An algorithm for automatic edge detection from magnetic anomalies

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Abstract

We propose an algorithm to estimate the depth and width of dyke-like magnetic sources, which is based on normalized vertical and total horizontal derivatives. The magnetic anomaly is firstly enhanced using an edge detection filter based on a simple transformation (the Signum transform) which retains only the signs of the anomalous field. Thus, the theoretical source locations, which rarely coincide with the real ones, are represented by positive plateaus. The theoretical edge positions can be recognized from the locations where one of the spatial field derivatives (or a function of them) change its sign: the zero crossover points. These zero crossover points are easily identified from the Signum transformed spatial derivatives. The actual sources depths and widths are then calculated using the distances between the center of the filtered anomalies and the zero crossover points, as input values. Our algorithm finds these distances by computing the radius of the largest circles within the theoretical sources locations, i.e., inside the region where the Signum transform equals 1. We validate this approach using a synthetic model and apply it to aeromagnetic data from Southern Brazil.

Introduction

Several methods have been proposed to estimate the location of potential field sources from gravimetric and magnetic data. In particular, methods based on field derivatives are easy to implement and demand a low computational effort, such as the tilt derivative (Miller and Singh, 1994; Verduzco et al., 2004), the Theta map (Wijns et al., 2005), the horizontal tilt derivative (TDX) (Cooper and Cowan, 2006), and the Tilt angle of the horizontal gradient (TAHG) (Ferreira et al., 2013).

Based on the fact that, for a vertical magnetization, the edges of the sources can be recognized from the locations where one or more of the spatial derivatives change its sign, de Souza and Ferreira (2012) proposed a simple derivative-based method, where these derivatives are normalized in order that only their signs (+1 or -1) are retained, namely the *Signum transform*. They showed that the points where the sign changes do not exactly coincide with the true edge location, and formulas for calculating the exact edge positions and depths for dyke-like sources were developed.

The formulas found in de Souza and Ferreira (2012) have not yet been fully employed because they depend on the distances from the center of the sources to the points where the sign of the Signum-transformed derivatives changes. Finding such distances from usual synthetic magnetic model data is straightforward (de Souza and Ferreira, 2013), but becomes challenging in the case of real data.

In this work we propose an algorithm that produces a map of these distances by computing the radius of the largest circles within the sources (where the Signum transform equals 1), from which the formulas from de Souza and Ferreira (2012) can be evaluated in an automatic fashion.

In the following, we review the underlying theory of the Signum transform and present the ideas of the distance-finding algorithm. Afterwards, we validate this approach with a synthetic model and use it to estimate the depth and width of the sources from aeromagnetic data acquired in the State of Santa Catarina, Southern Brazil.

Theory

We define the Signum transform of a function f as follows (de Souza and Ferreira, 2012):

$$ST[f](x) = \begin{cases} \frac{f(x)}{|f(x)|}, & f(x) \neq 0, \\ 1, & f(x) = 0 \end{cases} \quad (1)$$

When $f = M_z = \partial M/\partial z$, where M is the magnetic anomaly, the values of $ST[f]$ are expected to be 1 over the sources and -1 out of them. Thus, the sources are represented by plateaus where the value of the filtered anomalies is +1. However, the calculated boundaries are displaced from the real ones, especially for deep sources, making the bodies appear larger than they are.

In particular, the magnetic anomaly of a vertical dyke located at the magnetic pole and observed at $z = 0$ is

$$M = M(x,0) = A\left(\tan^{-1}\frac{x+a}{h} - \tan^{-1}\frac{x-a}{h}\right), \quad (2)$$

where A is the amplitude coefficient, a is one-half of dyke width and h is its depth. This expression is valid for anomalies observed near the center of a dyke of infinite length and depth. The magnetic anomaly in equation (2), as well as its first-order derivatives, are available in Murthy (1985).

The points x_v such that $M_z = M_z(x,0) = 0$ are given as

$$x_v = \pm\sqrt{a^2+h^2}, \quad (3)$$

which are obviously different from the actual edges $x = \pm a$. On the other hand, the roots of $M_z \pm |M_x|$ are given as

$$x_{vh-} = \pm(h - \sqrt{a^2+2h^2}), \quad x_{vh+} = \pm(h + \sqrt{a^2+2h^2}). \quad (4)$$

From the positive roots of x_v, x_{vh-}, and x_{vh+} we find the following expressions for h and a:

$$h = (x_{vh_+} - x_{vh_-})/2, \quad a = \sqrt{x_v^2 - h^2}. \tag{5}$$

Alternatively, the depth of the dyke may be computed as follows:

$$h = (x_v^2 - x_{vh_-}^2)/(2x_{vh_-}), \quad a = \sqrt{x_v^2 - h^2}. \tag{6}$$

These formulas are exact in the assumptions above, and provide estimates for dyke-like sources. To take into account variations in the y direction in three-dimensional anomalies, we replace in the above formulas M_x with M_h, where

$$M_h = \sqrt{M_x^2 + M_y^2} \tag{7}$$

is the total horizontal derivative.

Let us proceed to the computation of x_v, x_{vh-}, and x_{vh+} (for conciseness, we limit the details to x_v).

The fist step is to set $x_v = 0$ everywhere. Afterwards, for each grid point (x_i, y_i) where $ST[M_z](x_i, y_i) = 1$, we find the largest circle C_i such that $ST[M_z](x, y) = 1$ for any point $(x, y) \in C_i$. Defining r_i as the radius of C_i, we impose $x_v(x, y) = \max\{x_v(x, y), r_i\}$ for any $(x, y) \in C_i$, according to Figure 1.

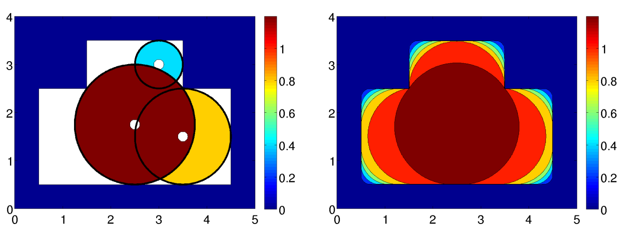

Figure 1: Illustration of the distance-finding algorithm. Left: algorithm action over the three points marked in white. At the points where the circles overlap, the radius of the larger circle (in brown) prevails. Right: final result of the algorithm, after all points in the inner region have been scanned.

From this procedure we have that x_v is nearly constant for thin rectangular (prismatic) bodies, as expected (Fig. 2).

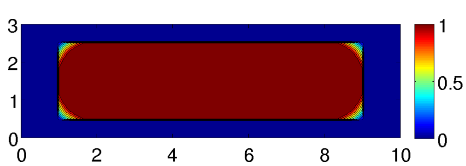

Figure 2: Final result of the distance-finding algorithm in over a thin rectangle. The distances are, almost everywhere, the height of the rectangle, as it would be expected from a rectangle with infinite length.

Synthetic example

Let us first consider a synthetic model shown in Figure 3, composed of two prisms. The prisms have the same length (4000 m) and thickness (4000 m), but different widths (P1 = 200 m and P2 = 400 m), with their tops located at depths 100 m (P1) and 300 m (P2). The magnetic anomaly in the pole, with induced magnetization only (J = 1.25 A/m), was generated with the open-source code Grav_Mag_Prism (Bongiolo et al., 2013) and is shown in Figure 4.

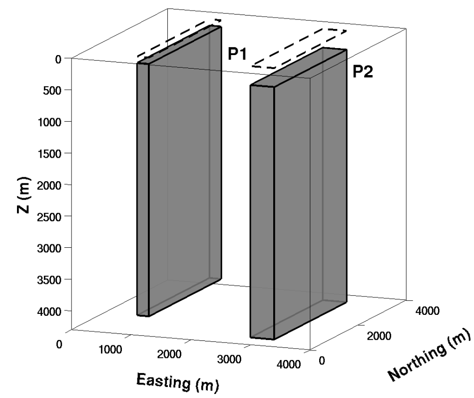

Figure 3: Synthetic 3D model of two prisms with dimensions 4000x200x4000 m (P1) and 4000x400x4000 m (P2), located at depths 100 m (P1) and 300 m (P2), and separated by distance of 2000 m.

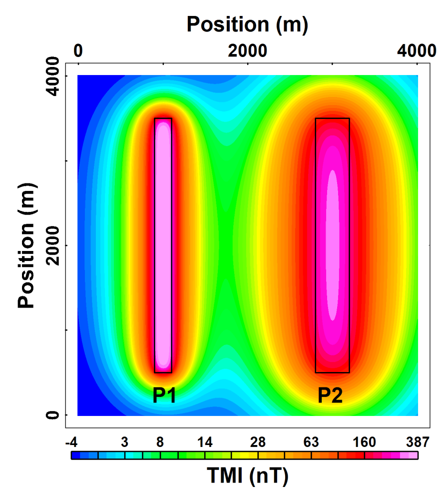

Figure 4: Magnetic anomaly map of the synthetic model shown in Figure 3.

We evaluated the Signum transforms of M_z and M_{zz} (Fig. 5) and $M_z \pm M_h$ (Fig. 6). The Signum transform of M_{zz} (Fig. 5, right) is not employed in formulas (5) and (6), but provides an accurate qualitative estimate for the horizontal locations of the sources.

From the Signum-transformed derivatives we evaluate x_v and $x_{vh\pm}$, and then compute h and a. The estimated depth and width of the prisms are shown in Figures 7 and 8, respectively, according to formulas (5) and (6). Although formula (6) provided a better estimate for the width of prism P2, according to Figure 8, formula (5) is numerically more stable and will be chosen for the next experiment.

Field example

In this section we use the algorithm as part of a study of aeromagnetic data from the State of Santa Catarina, southern Brazil (Fig. 9). The aeromagnetic

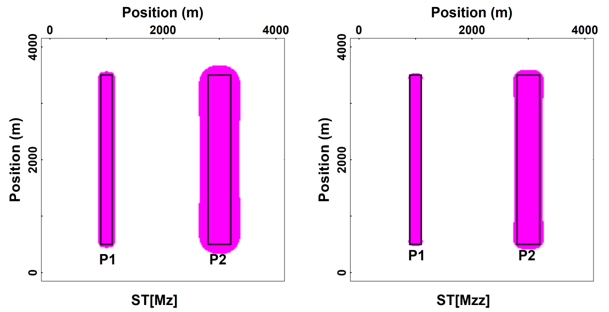

Figure 5: Signum transforms of M_z (left) and M_{zz} (right).

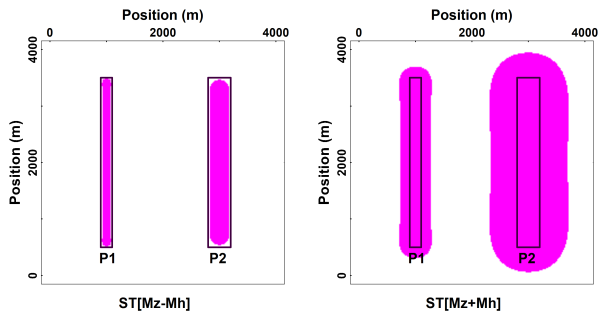

Figure 6: Signum transforms of $M_z - M_h$ (left) and $M_z + M_h$ (right), with $M_h^2 = M_x^2 + M_y^2$.

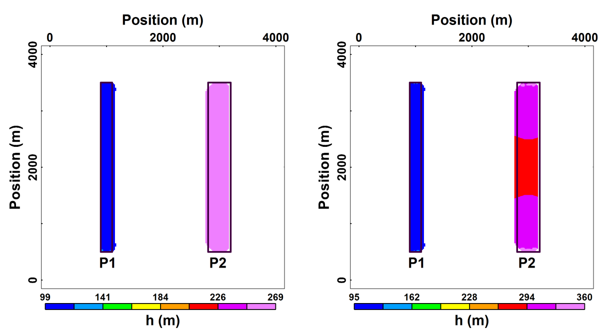

Figure 7: Depth of the prisms estimated from the Signum transforms from formula (5) (left) and formula (6) (right).

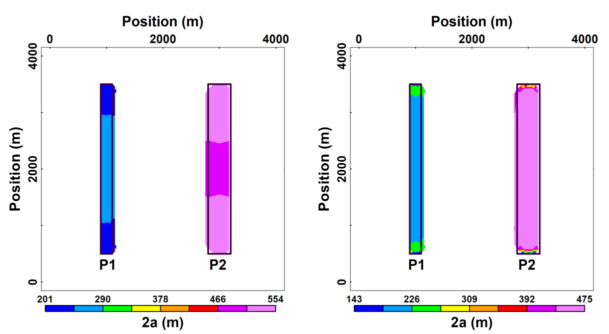

Figure 8: Width of the prisms estimated from the Signum transforms from formula (5) (left) and formula (6) (right).

data of the Paraná-Santa Catarina Project were acquired by Companhia de Pesquisa de Recursos Minerais (the Geologic Survey of Brazil) between 2009/11/10 and 2011/07/11 (CPRM, 2011), along north-south flight lines spaced at 500 m, with a mean terrain clearance at 100 m. The tie lines had 5 km spacing. The study area is located in the east of State of Santa Catarina, covering an area about 27.8 km by 27.8 km (see the inset of Fig. 9).

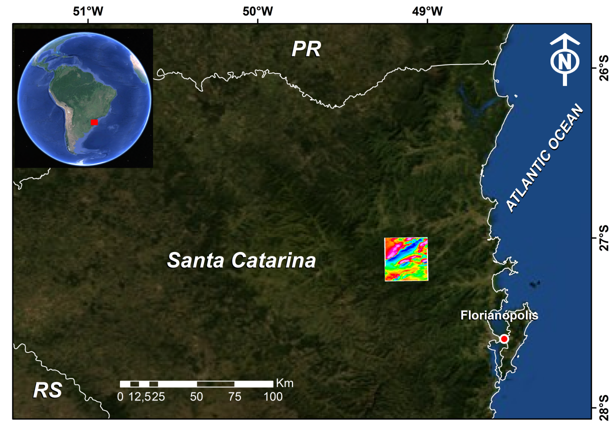

Figure 9: Location of the study area.

The TMI data (grid cell size of 100 m) in Figure 9 were reduced to the pole (RTP) using a magnetic inclination of -37.05^o and a declination of -18.17^o, which corresponds to the magnetic field at the time of the airborne survey in the study area (Fig. 10, left). Afterwards, the data were upward continued to 500 m to reduce the noise and aid delineation of deep structures (Fig. 10, right). The RTP magnetic maps (Fig. 10) show strong northeast-southwest trend to magnetic structures.

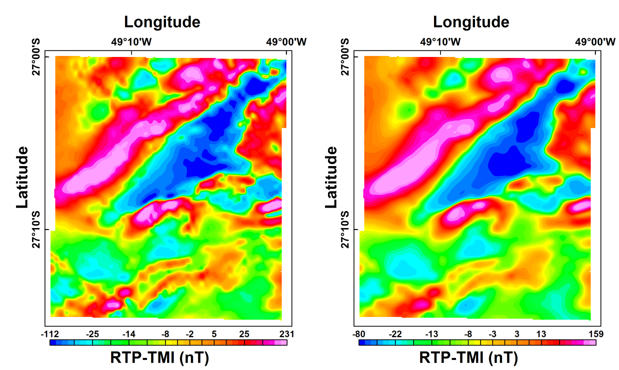

Figure 10: Left: RTP-TMI map of the magnetic anomalies shown in the inset of Figure 9. Right: RTP-TMI calculated from the same magnetic anomalies upward continued to 500 m.

Proceeding as in the synthetic model, we evaluate the Signum transforms of M_z, M_{zz}, and $M_z \pm M_h$, as shown in Figures 11 and 12.

The estimated depths of the sources, according to formula (5), are shown in Figure 13. We also compare these depths with the depths estimated by Euler deconvolution (Thompson, 1982) contact model solutions (SI = 0), and note that they qualitatively agree with each other.

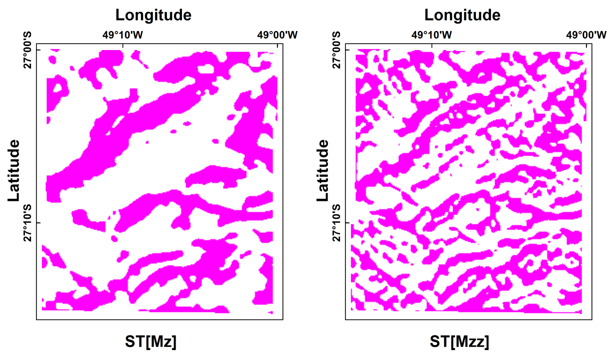

Figure 11: Signum transforms of M_z (left) and M_{zz} (right) from data shown in Figure 10 (right).

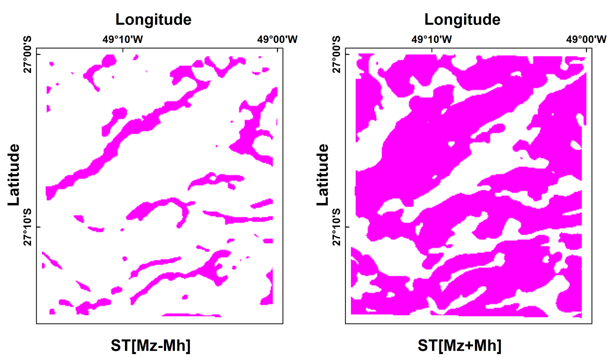

Figure 12: Signum transforms of $M_z - M_h$ (left) and $M_z + M_h$ (right), with $M_h^2 = M_x^2 + M_y^2$, from data shown in Figure 10 (right).

Figure 14 shows the corresponding widths also according to equation (5).

Conclusions

In this work we have extended the applicability of the Signum transform by providing estimates for the depth and width of linear sources. The discrepancies in the synthetic data between the depths and widths, calculated and observed, may be related to the assumptions of formula (2). Nevertheless, the algorithm provides initial estimates for these parameters, as illustrated in the similarity between the depth estimates from the Signum transform and the Euler deconvolution solutions.

Acknowledgments

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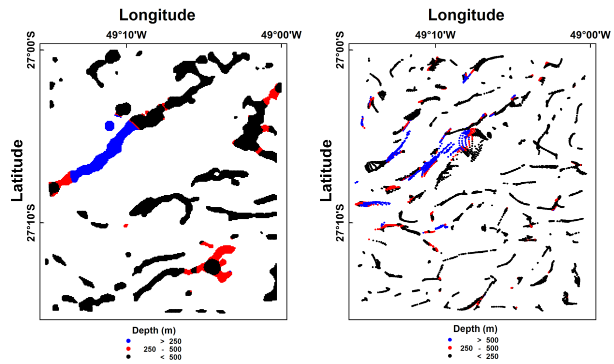

Figure 13: Depth of the sources estimated from the Signum transforms (left) and Euler deconvolution (right), from data shown in Figure 10 (right).

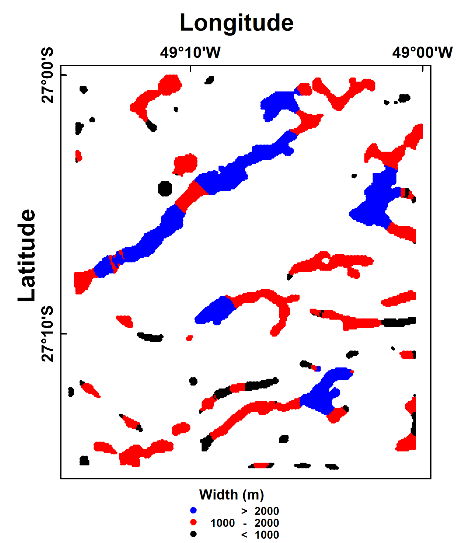

Figure 14: Width of the sources estimated from the Signum transforms, from data shown in Figure 10 (right).

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